Table 1 Geometric characteristics of four light-aircraft cabins and one representative of all four

Aircraft	Type	Cabin length, in.	R, in.	h (in.)	M	A_s (in.^{2})	I_{os} (in.^{4})	J_s (in.^{4})	$\bar{z}_s$ (in.)	A_r (in.^{2})	I_{or} (in.^{4})	J_r (in.^{4})	$\bar{z}_r$ (in.)
A	Single engine	112	25	0.030	33	0.032	0.006	0.0049	0.038	0.094	0.088	0.039	0.763
B	Single engine	110	25	0.030	22	0.065	0.015	0.0059	0.466	0.142	0.320	0.120	1.210
C	Twin engine	168	34	0.030	27	0.046	0.013	0.0078	0.413	0.165	0.264	0.102	1.013
D	Twin engine	127	31	0.040	30	0.035	0.003	0.0022	0.187	0.133	0.461	0.159	1.513
Representative	Various		32.0	0.040	30	0.046	0.013	0.0078	0.413	0.165	0.264	0.102	1.013

Table 2 Buckling analysis results

Subshell	$\bar{F}_{cr}$	n at $\bar{F}_{cr}$
I	1.33	15
II	2.25	15
III	2.66	15

Table 3 Influence of number of stringers on buckling load of SSII

Number of stringers, M	$\bar{F}_{cr}$
30	2.15
40	2.60
50	2.91
60	3.16

Table 4 Influence of spacing on buckling load

Dimensionless ring spacing, l/R	$\bar{F}_{cr}$
0.50	3.23
0.75	2.43
1.00	1.92
2.00	1.33

50% of each parameter, thus suggesting that stiffener eccentricity need not be considered. However, this is definitely not so, because if the stiffeners are placed on the outside rather than on the inside of the cylinder, the dynamic buckling load is increased by 28.2%.

The number of stiffeners, rather than the stiffener cross-sectional area, is the important design consideration, which is not surprising, in view of the local buckling behavior observed previously. The stiffeners can exert only a torsional restraint in the local buckling mode, so that "beefing up" the stiffeners affects the buckling load only slightly. An optimal aircraft fuselage design is suggested to be one in which the stiffener cross section is reduced until general instability is on the verge of predominating over local buckling. Then the reduction in stiffener weight could be used to increase the total number of stiffeners, resulting in a more crashworthy aircraft with no increase in weight.

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A Criterion for Assessing Wind-Tunnel Wall Interference at Mach 1

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Introduction

IN 1958 Spreiter and Alksne introduced the concept of local linearization to calculate pressures on airfoils for Mach numbers near one.[1] They subsequently extended the concept to bodies of revolution (or, for slender configurations, to equivalent bodies of revolution). A comprehensive review has been presented by Spreiter.[2] It will be shown how the same ideas can be used to determine a criterion for assessing wind tunnel wall interference. Using the criterion, it becomes possible, with the aid of the transonic similarity laws, to design transonic wind tunnel experiments that are virtually interference-free.

The Two-Dimensional Case

The flow is assumed to be inviscid and irrotational, and the free stream Mach number is unity. In that case, a perturbation velocity potential ϕ exists such that the axial perturbation velocity $u = \partial\phi/\partial x$, while the vertical perturbation velocity $w = \partial\phi/\partial z$. This velocity potential obeys the nonlinear partial differential equation

$$\frac{\partial^2\phi}{\partial z^2} = \frac{\gamma + 1}{U_\infty} \frac{\partial\phi}{\partial x} \frac{\partial^2\phi}{\partial x^2} \tag{1}$$

where x is the streamwise direction and z is normal to it.

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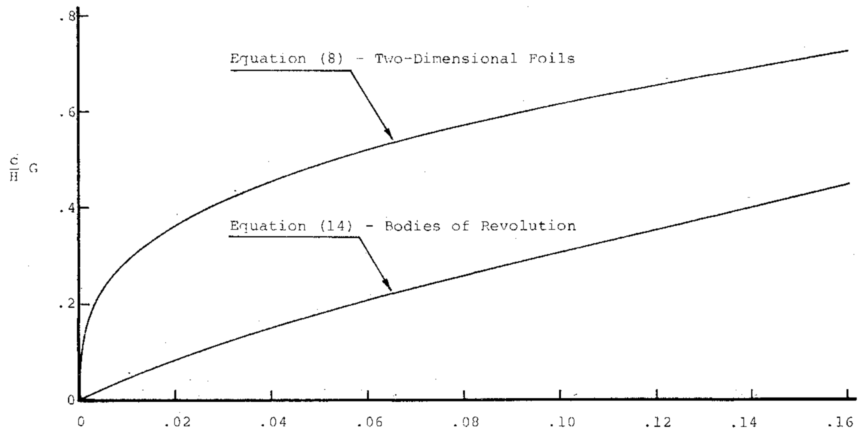

Fig. 1 Conditions for negligible interference at Mach 1.

Here U_∞ is the freestream velocity and γ is the ratio of specific heats. It is noted that the right hand side of this equation is nonlinear, and any analytical procedure must account for this nonlinearity. The boundary conditions require that the gradient of ϕ vanish far ahead of the body and that the flow be tangential to the body surface. For a thin airfoil lying along the x axis this is taken, as in linear theory, to be

$$\partial\phi/\partial z\,|_{z=0} = U_\infty\,\alpha \tag{2}$$

where α is the local slope of the foil.

In the method of local linearization Eq. (1) is replaced by

$$\partial^2\phi/\partial z^2 = \lambda\partial\phi/\partial x \tag{3}$$

where

$$\lambda = (\gamma + 1/U_\infty)\partial u/\partial x > \bigcirc \tag{4}$$

and in the early stages of the analysis γ is taken to be constant. In that case Eq. (3) is the equation for one-dimensional transient heat conduction, and at distances

$$z \geq \bigcirc\left(\frac{x}{\lambda}\right)^{1/2}$$

The solution will become vanishingly small. Hence, if a wind tunnel wall were to be situated at such a distance, where x is the chord of the foil, its effect on the foil would be negligible. Thus, we obtain the condition for negligible wall interference:

$$H(\lambda/c)^{1/2} > \bigcirc(1) \tag{5}$$

where H is the half-height of the tunnel and c is the chord of the foil.

The quantity λ is not, of course, known, but from the transonic similarity laws it is known that[2]

$$u/U_\infty = \bigcirc[\tau^{2/3}/(\gamma + 1)^{1/3}] \tag{6}$$

where τ is the thickness ratio of the airfoil. Hence, from Eq. (4),

$$\lambda = \bigcirc[\tau^{2/3}(\gamma + 1)^{2/3}]/c \tag{7}$$

Upon substituting into Eq. (5) it is found that the condition for negligible wall interference becomes

$$G \equiv (H/c)\tau^{1/3}(\gamma + 1)^{1/3} \geq \bigcirc(1) \tag{8}$$

Thus, a means is presented for ordering various tests according to the amount of wall interference to be expected, since the larger the value of G the less the interference will be. For those tests for which G becomes of order one or greater, the interference may be presumed to be negligible. A plot of $G(c/H)$ vs τ for $\gamma = 1.4$ is shown in Fig. 1.

It can be seen immediately that the condition given in Eq. (8) is paradoxical in that it indicates that wall interference can be made negligible either by increasing the size of the tunnel with respect to the chord of the foil or by increasing the thickness ratio of the foil, though the former means is far more effective than the latter. The reason that increasing the thickness ratio of the foil is effective as a means to reach the stage where wall interference is negligible can perhaps best be understood for a foil whose rear portion is locally at supersonic speed. The flow near the rear of the thicker foil will reach a higher Mach number than the flow near the rear of the thinner foil. As a consequence, the waves reflected from the tunnel walls will bend more for the thicker foil than for the thinner foil, and so, all other things being equal, the first reflected wave will impinge on the foil somewhat farther downstream for the thicker foil than for the thinner foil. Thus, the possibility of this wave missing the foil altogether is greater for the thicker foil than for the thinner foil. Even for flows that are completely subsonic over the foil (such as the flow over a finite wedge) the rule given above applies, though the physical explanation is more obscure.

At this writing, there does not appear to be any experimental evidence either to confirm or to deny the order of magnitude relationship given by Eq. (8). However, using the hodograph method Marschner[3] has determined the correction to the freestream pressure coefficient on a wedge in a sonic freejet, and from his Eq. (9.14b) it can readily be shown that the correction term is negligible compared with the free air term provided a condition of precisely the form of Eq. (8) is satisfied. From the numerical point of view, Marschner chose to illustrate the smallness of his correction for $\tau = 0.1$, $c/H = 0.13$, $\gamma = 1.4$. For these values $G = 4.8$ which is certainly at least of order one.

Indeed, it is difficult to envision a test for which G would not be of order one. As can be seen from Fig. 1 the quantity (c/H)G itself is at least 0.5 for all foils with a thickness ratio greater than 5%, so that the wind-tunnel walls do not have to be very far away from the foil in order to affect negligible interference. This situation does not prevail for a body of revolution, however, as will now be demonstrated.

Axially Symmetric Case

For bodies of revolution Eq. (3) is replaced by

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\phi}{\partial r}\right) = \lambda\frac{\partial\phi}{\partial x} \tag{9}$$

which is the equation for transient heat conduction with cylindrical symmetry. At distances

$$r \geq \bigcirc(x/\lambda)^{1/2}$$

the solution will become vanishing small, and hence we obtain the condition for negligible wall interference;

$$H(\lambda/c)^{1/2} \geq O(1) \tag{10}$$

where, in this case, H is the distance from the model to the nearest wall of the wind tunnel. (If the tunnel were circular and the model were at its center, H would be the tunnel radius.)

The axial velocity is known from the similarity laws for transonic flow to obey the relationship[2]

$$\frac{u}{U_\infty} = O[\tau^2 \ln(\tau^2(\gamma + 1)^{1/2})] + O(\tau^2) \tag{11}$$

Neglecting the term $O(\tau^2)$ and substituting into Eq. (4) it is found that

$$\lambda = O\left[\frac{(\gamma + 1)\tau^2 \ln(\tau^2(\gamma + 1)^{1/2})}{c}\right] \tag{13}$$

Upon substituting into Eq. (10) it is found that the condition for negligible wall interference becomes

$$G \equiv \frac{H}{c}(\gamma + 1)^{1/2}\tau\left|\ln\tau^2(\gamma + 1)^{1/2}\right|^{1/2} \geq O(1) \tag{14}$$

Once again G serves as a means for ordering various tests according to the amount of wall interference to be expected. A plot of $G(c/H)$ vs τ for $\gamma = 1.4$ is shown in Fig. 1. Once again, it is seen that increasing the thickness ratio tends to reduce wall interference. But for the axially symmetric case the quantity $G(c/H)$ is considerably smaller than for the two-dimensional case so that a larger tunnel is required to achieve negligible interference.

The reason the axially symmetric case is more critical than the two-dimensional case with respect to wind tunnel wall interference is probably because the increase in Mach number over a body of a given thickness ratio is less for the axially symmetric case than for the two-dimensional case; as a consequence, the waves in the supersonic portion of the flow do not bend as much for the axially symmetric case and the reflected waves are, therefore, more prone to impinge on the model.

It is instructive to calculate G for some early transonic tests on bodies of revolution that were carried out in the 1950's.

Spreiter[2] has presented experimental data for various bodies of revolution tested in various transonic wind tunnels by several different investigators and compared the results with the theoretical results obtained by the method of local linearization.

Page[4] tested a cone-cylinder having a half cone angle of 7° and a cone length of 5.5 in. in two different tunnels at the Ames Research Center. One tunnel was 2 ft square, and the other was 14 ft square. The corresponding values of G are 1.27 and 8.89 respectively. Data for the value of G is in better agreement with Spreiter's approximate theory, though there are still considerable discrepancies.

Drougge[5] tested a parabolic arc body of revolution with thickness ratio 1/6 having a maximum diameter of 20 × $(2)^{1/2}$ mm. in a wind tunnel 90 cm, square. The value of G was 1.24. A geometrically similar body having a maximum diameter of 10 in. was tested in the Ames 14 ft wind tunnel. The value of G was 0.65. Another parabolic arc body of revolution having a maximum diameter of 6 in. and a thickness ratio of 1/12 was also tested in the 14 ft wind tunnel. The value of G was 0.31.

For all the parabolic arc bodies of revolution the agreement between theory and experiment worsened as G became smaller. The value of G for Page's experiment on the cone cylinder in the 2 ft tunnel was approximately equal to the value of G for the parabolic arc body tested by Drougge. But the agreement between theory and experiment was considerably better for Drougge's experiments than for Page's. Spreiter states that wall interference effects are particularly severe on the cone-cylinder and we agree. We also note that the two-dimensional analogy of the cone, viz. the wedge, may be expected to experience larger interference than the two-dimensional analogy of the parabolic arc body of revolution viz., the circular arc airfoil. Indeed, using an integral method, Goodman[6] has shown how to calculate a critical value of G for various affinely similar two-dimensional shapes. $G_{\text{crit.}}$ is defined such that if $G > G_{\text{crit.}}$ the interference is virtually negligible. For the wedge $G_{\text{crit.}} = 2.22$, while for the circular arc airfoil $G_{\text{crit.}} = 1.20$, which confirms that the wedge is more subject to interference than the circular arc airfoil. No method has yet been devised for determining $G_{\text{crit.}}$ for bodies of revolution.

Recommendations

In order to design experiments that are relatively interference-free it is recommended that the model be made as short as possible (small c) which is obvious. What is not obvious is that in order to obtain interference-free data it may be possible to test an affinely similar model with a larger thickness ratio than the full-scale body that the model is meant to represent, and then to calculate the desired aerodynamic quantities for the actual body using the transonic similarity rules. This concept would be of most significance in the testing of slender bodies. It is hoped that the idea will be tried by experimenters in the near future and that data obtained in this way will be compared with free flight data, with theory, and with wind-tunnel data obtained using a model that is geometrically similar to the full-scale body.

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